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# Lepton masses and mixing angles with spontaneously broken O(3) flavor symmetry

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Abstract: We present a model based on an O(3) flavor symmetry and a minimal extension of the scalar sector to induce hierarchical breaking, $\mathrm{O}(3) \to \mathrm{O}(2) \to \mathrm{SO}(2) \to$ nothing. The model naturally accounts for all the known lepton parameters and yields various interesting predictions for others: (i) Neutrinos are nearly degenerate, $m_\nu \sim 0.1\,\mathrm{eV}$; (ii) The solar neutrino problem is solved by the MSW small mixing angle solution; (iii) The MNS mixing angle $\theta_{13}$ is unobservably small, $\theta_{13} = \mathcal{O}(10^{-5})$.

Keywords: Beyond Standard Model, Neutrino Physics, Phenomenological Models.

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## Contents



## 1. Introduction

The SuperKamiokande atmospheric neutrino experiment [1] has provided evidence for neutrino masses. There have been many attempts to understand the lepton flavor pattern in the framework of flavor symmetries. Models with an O(3) or SO(3) flavor symmetry [2] are particularly interesting due to the fact that the O(3) symmetry naturally accommodates degenerate neutrinos. The possibility of nearly degenerate neutrinos is in agreement with all the experimental data and is motivated by the possibility that neutrinos play a role in the evolution of the large-scale structure of the universe (see e.g. [3] and references therein).

In this work we present an economical model based on a spontaneously broken O(3) flavor symmetry, which naturally satisfies the constraints from the experimental data and yields several non-trivial predictions for upcoming neutrino experiments.

In order to make our discussion concrete we briefly review the experimental data related to the lepton sector. In an effective two neutrino framework, the data from the atmospheric neutrino experiments leads to the following results (at 99% CL) [4]:

$$\Delta m^2_{23} \sim (1-8)\times 10^{-3}\,\mathrm{eV}^2\,, \qquad \sin^2 2\theta_{23} > 0.85\,. \tag{1.1}$$

In a two generation framework the solar neutrino experimental data [5] yield several viable solutions for the mass difference and mixing angle [4] (at 99% CL):

i. The large mixing angle (LMA) MSW [6] solution:

$$\Delta m^2_{12} \sim 10^{-5} - 10^{-3}\,\mathrm{eV}^2\,, \qquad \tan^2\theta_{12} \sim 0.1 - 1\,. \tag{1.2}$$

ii. The large mixing angle with a low mass-squared difference (LOW) and the quasi-vacuum oscillation (QVO) solution:

$$\Delta m^2_{12} \sim 5\times 10^{-10} - 3\times 10^{-7}\,\mathrm{eV}^2\,, \qquad \tan^2\theta_{12} \sim 0.4 - 3\,. \tag{1.3}$$

iii. The small mixing angle (SMA) MSW solution:

$$\Delta m^2_{12} \sim (3-10)\times 10^{-6}\,\mathrm{eV}^2\,, \qquad \tan^2\theta_{12} \sim 2\times 10^{-4} - 2\times 10^{-3}\,. \tag{1.4}$$

Combining the CHOOZ experiment results [7] with the solar and the atmospheric neutrino experiments yields the following constraints, in a three generation framework:

$$\Delta m^2_{23} \sim (1-7)\times 10^{-3}\,\mathrm{eV}^2\,, \qquad \sin^2\theta_{13} < 0.075\,. \tag{1.5}$$

The experimental data [8] from neutrinoless double beta decay indicates that (at 90% CL):

$$(M_\nu)_{ee} < 0.2\,\mathrm{eV}\,, \tag{1.6}$$

where $(M_\nu)_{ee}$ is the value of the (11) element of the neutrinos mass matrix in the basis where both the charged lepton mass matrix and the weak interaction couplings are diagonal. Finally the charged lepton masses are given by [9]:

$$\begin{aligned} m_e &\simeq 0.51\,\mathrm{MeV}\,, \\ m_\mu &\simeq 105.7\,\mathrm{MeV}\,, \\ m_\tau &\simeq 1777\,\mathrm{MeV}\,. \end{aligned} \tag{1.7}$$

## 2. The model

We consider an effective theory with a cut-off scale $M$. We include nonrenormalizable terms induced by the integration out of the heavy degrees of freedom, with masses larger than $M$.

The lagrangian is invariant under an O(3) flavor symmetry. The field content consist of the SM fields and additional SM gauge singlet scalar fields. The additional fields are $S^{ij}$, $i, j \in \{1\ldots 3\}$, a traceless symmetric field which transforms as a **5** of the O(3) symmetry; $\Phi^j$, a triplet of the O(3) symmetry and $A$, a pseudo-singlet of the O(3) symmetry. The lepton $\mathrm{SU}(2)_L$ doublets, $L^i$, transform as a triplet of the flavor group. Two of the right handed charged leptons, $E^2_R$ and $E^3_R$, are singlets of the O(3) symmetry, while the third one, $E^1_R$, is a pseudo-singlet.

The masses and the mixing angles of the leptons are induced by the nonzero VEVs of the scalar fields. We assume that CP is conserved in the lepton sector. Therefore the couplings in the lagrangian and the VEVs of the scalars are real. The flavor symmetry is broken by three small parameters, $\delta_{1,2}$ and $\epsilon$, as follows:

$$\mathrm{O}(3) \;\xrightarrow[\delta_1]{\langle S^{ij}\rangle}\; \mathrm{O}(2) \;\xrightarrow[\delta_2]{\langle A\rangle}\; \mathrm{SO}(2) \;\xrightarrow[\epsilon\delta_1]{\langle \Phi^i\rangle}\; \emptyset\,, \tag{2.1}$$

with

$$\begin{aligned}\frac{\langle S^{ij}\rangle}{M} &= \delta_1\cdot \mathrm{diag}(1,1,-2)\,,\\ \frac{\langle A\rangle}{M} &= \delta_2\,,\\ \frac{\langle \Phi^i\rangle}{M} &= \epsilon\delta_1\cdot(0,\sin\alpha,\cos\alpha)\,.\end{aligned} \tag{2.2}$$

## 2.1 The neutrino mass matrix

Neutrino masses are related to the following terms in the lagrangian:

$$\mathcal{L}_\nu = \left\{L^iL^i + \frac{1}{M}aL^iL^jS^{ij} + \frac{1}{M^2}[b(L^i\Phi^i)^2 + b'L^jL^j\Phi^i\Phi^i]\right\}\frac{HH}{M} + \mathrm{h.c.}\,, \tag{2.3}$$

where $H$ is the Higgs field and the coefficients $a, b, b'$ are of order unity. Given the breaking pattern of eq. (2.1), the lagrangian in eq. (2.3) induces the following neutrino mass matrix:

$$M_\nu = m\begin{pmatrix} 1+a\delta_1+b'\delta_1^2\epsilon^2 & 0 & 0\\ 0 & 1+a\delta_1+b'\delta_1^2\epsilon^2+b\delta_1^2\epsilon^2s^2 & b\delta_1^2\epsilon^2sc\\ 0 & b\delta_1^2\epsilon^2sc & 1-2a\delta_1+b'\delta_1^2\epsilon^2+b\delta_1^2\epsilon^2c^2\end{pmatrix}. \tag{2.4}$$

with $m = \langle H\rangle^2/M$, $s=\sin\alpha$ and $c=\cos\alpha$.

The mixing angles $\theta^\nu_{ij}$ required to diagonalize the mass matrix in eq. (2.4) are given by:

$$\begin{aligned}\tan 2\theta^\nu_{23} &\approx 2\frac{\delta_1\epsilon^2bsc}{3a+\delta_1\epsilon^2b(s^2-c^2)} \ll 1\,,\\ \theta^\nu_{13}\,,\theta^\nu_{12} &\approx 0\,.\end{aligned} \tag{2.5}$$

Subleading corrections to eq. (2.5) appear when higher dimension operators such as $\epsilon^{ijk}AL^m\Phi^m\Phi^nS^{ni}\Phi^jL^kHH/M^6$ are added to the lagrangian in eq. (2.3).

The eigenvalues of (2.4) are given, to leading order, by:

$$\begin{aligned}m_{\nu_1} &= m(1+a\delta_1+b'\delta_1^2\epsilon^2)\,,\\ m_{\nu_2} &= m(1+a\delta_1+b'\delta_1^2\epsilon^2+b\delta_1^2\epsilon^2s^2)\,,\\ m_{\nu_3} &= m(1-2a\delta_1+b'\delta_1^2\epsilon^2+b\delta_1^2\epsilon^2c^2)\,.\end{aligned} \tag{2.6}$$

Since the contributions to the mixing angles from the neutrino sector in eq. (2.5) are negligibly small, the mixing angles of the MNS matrix [10] will be determined by the charged lepton mass matrix.



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### 2.2 Charged leptons mass matrix

Charged lepton masses are related to the following terms in the lagrangian:

$$\begin{aligned}\mathcal{L}_\ell = & \left\{ \left( a_3 L^{i\dagger}\Phi^i + b_3 \frac{1}{M} L^{i\dagger} S^{ij}\Phi^j \right) E_R^3 + \left( a_2 L^{i\dagger}\Phi^i + b_2 \frac{1}{M} L^{i\dagger} S^{ij}\Phi^j \right) E_R^2 + \right. \\ & + \frac{A}{M}\left[ \left( a_1 L^{i\dagger}\Phi^i + b_1 \frac{1}{M} L^{i\dagger} S^{ij}\Phi^j \right) E_R^1 + d_3 \frac{1}{M^2}\epsilon^{ijk} L^{i\dagger}\Phi^l S^{lj}\Phi^k E_R^3 + \right. \\ & \left. \left. + \, d_2 \frac{1}{M^2}\epsilon^{ijk} L^{i\dagger}\Phi^l S^{lj}\Phi^k E_R^2 \right] + d_1 \frac{1}{M^2}\epsilon^{ijk} L^{i\dagger}\Phi^l S^{lj}\Phi^k E_R^1 \right\} \frac{H}{M} + \text{h.c.}\,, \end{aligned} \tag{2.7}$$

where the coefficients $a_i, b_i$ and $d_i$ are of order unity. Given (2.7) and the breaking pattern of eq. (2.1) the following mass matrix is obtained:

$$M_\ell = m_\ell \begin{pmatrix} 3d_1 sc\delta_1^2\epsilon & 3d_2 sc\delta_2\delta_1^2\epsilon & 3d_3 sc\delta_2\delta_1^2\epsilon \\ s\delta_2(a_1 + b_1\delta_1) & s(a_2 + b_2\delta_1) & s(a_3 + b_3\delta_1) \\ c\delta_2(a_1 - 2b_1\delta_1) & c(a_2 - 2b_2\delta_1) & c(a_3 - 2b_3\delta_1) \end{pmatrix}, \tag{2.8}$$

where $m_\ell = \langle H \rangle \delta_1 \epsilon$.

As we saw, the mixing angles in the neutrino sector are negligibly small (2.5). Therefore, to leading order, the mixing angles required to diagonalize the charged lepton mass matrix in eq. (2.8) will determine the mixing angles of the MNS matrix. Thus the angle $\theta_{23}$ of the MNS matrix is given by:

$$\tan 2\theta_{23} \approx -\tan 2\alpha + \mathcal{O}\left(\delta_1, \delta_2^2\right) . \tag{2.9}$$

It is of order unity, in agreement with the experimental data eq. (1.1). The mixing angle $\theta_{13}$ is given by:

$$\tan 2\theta_{13} = \mathcal{O}\left(\epsilon\delta_2\delta_1^2\right) \ll 1\,, \tag{2.10}$$

in agreement with eq. (1.5). The angle $\theta_{12}$ is:

$$\tan 2\theta_{12} = \mathcal{O}(\epsilon\delta_2) \ll 1\,. \tag{2.11}$$

The eiganvalues of (2.8) are given, to leading order, by

$$\begin{aligned} m_e &= 3 m_\ell d_1 \delta_1^2 \epsilon s c\,, \\ m_\mu &= 3 m_\ell \delta_1 s c \frac{(a_3 b_2 + a_2 b_3)}{a_{23}}\,, \\ m_\tau &= m_\ell a_{23}\,, \end{aligned} \tag{2.12}$$

where $a_{23} \equiv \sqrt{a_2^2 + a_3^2}$. The relations in eq. (2.12) are valid as long as $\delta_2 \lesssim \delta_1$.

### 2.3 Consistency and predictions

In the previous subsections (2.1) and (2.2) we found expressions for the six masses and three mixing angles — the flavor parameters of the lepton sector. Using the experimetal data given in the first section, we can constrain the four free parameters $\delta_{1,2}, \epsilon, M$, test the model and identify its predictions for the upcoming experiments.

### 2.3.1 Constraining the small parameters

The charged lepton masses are known rather accurately, consequently they provide stringents constraint on the free parameters. The known ratio between the muon and the tau masses eq. (1.7) and the corresponding predicted ratio given in eq. (2.12) give:

$$\frac{m_\mu}{m_\tau} \approx 3\delta_1 sc\frac{a_3b_2 + a_2b_3}{a_{23}^2} \quad\Longrightarrow\quad \delta_1 = \mathcal{O}\left(0.03\right) . \tag{2.13}$$

From eqs. (1.7),(2.12) and (2.13) we further find:

$$\frac{m_\tau}{\langle H\rangle} \approx \delta_1\epsilon a_{23} \quad\Longrightarrow\quad \epsilon = \mathcal{O}\left(0.3\right) . \tag{2.14}$$

In order to set the allowed range of $\delta_2$ and $M$ we turn to the neutrino sector. Eq. (2.11) implies that the only possible solution to the solar neutrino problem in our model is the SMA solution. Using eqs. (1.4), (2.11) and (2.14) we get:

$$\tan\theta_{12} = \mathcal{O}\left(\epsilon\delta_2\right) \quad\Longrightarrow\quad \delta_2 = \mathcal{O}\left(0.1 - 0.01\right) . \tag{2.15}$$

Using eqs. (1.4), (2.6), (2.13) and (2.14) we get:

$$\Delta m_{12}^2 \sim 2bs^2m^2\delta_1^2\epsilon^2 \quad\Longrightarrow\quad m = \mathcal{O}\left(0.1 \text{ eV}\right) \quad\Longrightarrow\quad M = \mathcal{O}\left(10^{14} \text{ GeV}\right) . \tag{2.16}$$

Note that the large scale of $M$, or equivalently of $\langle S^{ij}\rangle$ and $\langle\phi^i\rangle$, makes any process related to the correponding massless Goldstone bosons practically unobservable [11].

### 2.3.2 Consistency checks

The lepton sector contains nine CP conserving flavor parameters, constrained by the experimental data as given in eq. (1.1) and eqs. (1.4)-(1.7). Four of them were used to find the values of the model free parameters. This implies that there are five more relations that can be compared with the corresponding experimental data and either test the model or give predictions. We have already pointed that the mixing angles $\theta_{23}$ and $\theta_{13}$ given in eqs. (2.9) and (2.10) satisfy the constraints in eqs. (1.1) and (1.5). There are three more non-trivial consistency checks:

(i) The well known ratio between the electron and muon masses:

$$\frac{m_e}{m_\mu} \approx \delta_1\epsilon\frac{d_1a_{23}}{a_3b_2 + a_2b_3} = \mathcal{O}\left(10^{-2}\right), \tag{2.17}$$

is consistent with eq. (1.7).

(ii) The neutrinos mass-squared difference between the second and third generation:

$$\Delta m_{23}^2 \approx 6m^2a\delta_1 = \mathcal{O}\left(10^{-3} \text{ eV}^2\right), \tag{2.18}$$

is consistent with eq. (1.1).

(iii) The bound from neutrinoless double beta decay translates into a constraint on $(M_\nu)_{ee}$, defined below eq. (1.6). In our case it is well approximated by $(M_\nu)_{11}$ which was calculated in eq. (2.4). Hence:

$$(M_\nu)_{11} \sim m = \frac{\langle H\rangle^2}{M} = \mathcal{O}\left(10^{-1}\ \mathrm{eV}\right), \tag{2.19}$$

is consistent with eq. (1.6).

To explicitly demonstrate the phenomenological consistency of our model, we set numerical values (the numbers are not necessarily the most favorable ones) to $\delta_i, \epsilon$ and $M$:

$$\begin{aligned} \delta_1 &= 0.035\,, \\ \delta_2 &= 0.03\,, \\ \epsilon &= 0.26\,, \\ M &= 1.5 \cdot 10^{14}\,\mathrm{GeV}\,. \end{aligned} \tag{2.20}$$

Substituting the numerical values for the above parameters yields the following values for the different observables:

$$\begin{aligned} \left[\frac{m_e}{0.51\,\mathrm{Mev}}\right] &\simeq 1.5 \cdot d_1 \sin 2\alpha\,, \\ \left[\frac{m_\mu}{106\,\mathrm{Mev}}\right] &\simeq 0.8 \cdot \frac{a_3 b_2 + a_2 b_3}{a_{23}} \sin 2\alpha\,, \\ \left[\frac{m_\tau}{1780\,\mathrm{Mev}}\right] &\simeq 0.9 \cdot a_{23}\,, \\ (M_\nu)_{ee} &\sim m \simeq 0.2\,\mathrm{eV}\,, \\ \Delta m_{12}^2 &\simeq 7 \cdot 10^{-6} \cdot b \sin^2\alpha\,\mathrm{eV}^2\,, \\ \Delta m_{23}^2 &\simeq 9 \cdot 10^{-3} \cdot a\,\mathrm{eV}^2\,, \\ \tan\theta_{12} &= \mathcal{O}(\epsilon\delta_2) \sim 10^{-2}\,. \end{aligned} \tag{2.21}$$

Taking into account the unknown coefficient of order one, eq. (2.21) fit reasonably well the experimental data.

### 2.3.3 Predictions

The model contains several non trivial predictions:

- The neutrinos are nearly degenerate with mass $m$, $m = \mathcal{O}\left(0.1\,\mathrm{eV}\right)$.

- The correct solution of the solar problem is the SMA, with the ratio between the square mass difference $\Delta m_{12}^2$ and $\Delta m_{23}^2$ given by:

$$\frac{\Delta m_{12}^2}{\Delta m_{23}^2} = \mathcal{O}\left(10^{-3}\right). \tag{2.22}$$

- The ratio between the small mixing angles $\theta_{13}$ and $\theta_{12}$ is given by:

$$\frac{\theta_{13}}{\theta_{12}} = \mathcal{O}\left(10^{-3}\right) . \tag{2.23}$$

## 3. Summary and conclusion

We presented a model with an O(3) flavor symmetry that is spontaneously broken by hierarchical VEVs of scalars, O(3) $\to$ O(2) $\to$ SO(2) $\to$ nothing. The lepton flavor parameters were derived from the most general lagrangian consistent with the symmetry and taking all dimensionless couplings to be of order unity. The model naturally accounts for all the known lepton parameters, passes several consistency checks and yields various interesting predictions: (i) Neutrinos are nearly degenerate, $m_\nu \sim 0.1\,\text{eV}$. (ii) The solar neutrino problem is solved by the MSW small mixing angle solution. (iii) The MNS mixing angle $\theta_{13}$ is unobservably small, $\theta_{13} = \mathcal{O}(10^{-5})$.

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## References

[1] Super-Kamiokande collaboration, *Measurement of a small atmospheric $\nu_\mu/\nu_e$ ratio*, *Phys. Lett.* **B 433** (1998) 9 [hep-ex/9803006]; *Study of the atmospheric neutrino flux in the multi-GeV energy range*, *Phys. Lett.* **B 436** (1998) 33 [hep-ex/9805006]; *Evidence for oscillation of atmospheric neutrinos*, *Phys. Rev. Lett.* **81** (1998) 1562 [hep-ex/9807003]; *Measurement of the flux and zenith-angle distribution of upward through-going muons by* Super-Kamiokande, *Phys. Rev. Lett.* **82** (1999) 2644 [hep-ex/9812014].

[2] See e.g. R. Barbieri, L.J. Hall, G.L. Kane and G.G. Ross, *Nearly degenerate neutrinos and broken flavour symmetry*, hep-ph/9901228;
C. Wetterich, *Natural maximal $\nu_\mu\nu_\tau$ mixing*, *Phys. Lett.* **B 451** (1999) 397 [hep-ph/9812426];
C.D. Carone and M. Sher, *Supersymmetric model of quasi-degenerate neutrinos*, *Phys. Lett.* **B 420** (1998) 83 [hep-ph/9711259];
A. Ghosal, *Bi-maximal neutrino mixing with* SO(3) *flavour symmetry*, hep-ph/9905470;
M. Tanimoto, T. Watari and T. Yanagida, *Democratic mass matrices from broken* $\mathrm{O}(3)_L \times \mathrm{O}(3)_R$ *flavor symmetry*, *Phys. Lett.* **B 461** (1999) 345 [hep-ph/9904338];
Y.-L. Wu, *Two/three-flavor oscillation and MSW/just-so solution of neutrinos in the* SO(3) *gauge model*, *Int. J. Mod. Phys.* **A 14** (1999) 4313 [hep-ph/9901320];



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Y.-L. Wu, SO(3) *gauge model for neutrino masses and oscillations*, *Nucl. Phys.* **85** *(Proc. Suppl.)* (2000) 193 [hep-ph/9908436];
Y.-L. Wu, *Nearly degenerate mass and bi-maximal mixing of neutrinos in the* SO(3) *gauge model of leptons*, *Eur. Phys. J.* **C 10** (1999) 491 [hep-ph/9901245];
Y.-L. Wu, SO(3) *gauge symmetry and neutrino-lepton flavor physics*, *Phys. Rev.* **D 60** (1999) 073010 [hep-ph/9810491];
E. Ma, *Splitting of three nearly mass-degenerate neutrinos*, *Phys. Lett.* **B 456** (1999) 48 [hep-ph/9812344];
P. Bamert and C.P. Burgess, *Naturally degenerate neutrinos*, *Phys. Lett.* **B 329** (1994) 289 [hep-ph/9402229];
A.S. Joshipura, *Almost degenerate neutrinos*, *Z. Physik* **C 64** (1994) 31.

[3] J.R. Primack and M.A. K. Gross, *Cold + hot dark matter after* Super-Kamiokande, in Proceedings of the Xth Rencontres de Blois, *The birth of galaxies*, 28 June – 4, July, 1998 astro-ph/9810204;
J.R. Primack, J. Holtzman, A. Klypin and D.O. Caldwell, *Cold + hot dark matter cosmology with $m(\nu_\mu) \approx m(\nu_\tau) \approx 2.4\,eV$*, *Phys. Rev. Lett.* **74** (1995) 2160 [astro-ph/9411020].

[4] M.C. Gonzalez-Garcia, *Solar and atmospheric neutrino oscillations* parallel talk in *ICHEP 2000* Osaka, Japan, July 28, 2000 hep-ph/0010136;
M.C. Gonzalez-Garcia and C. Peña-Garay, *Global and unified analysis of solar neutrino data*, hep-ph/0009041.

[5] R. Davis, *A review of the homestake solar neutrino experiment Prog. Part. Nucl. Phys.* **32** (1994) 13;
Y. Fukuda et al., *Solar neutrino data covering solar cycle 22*, *Phys. Rev. Lett.* **77** (1996) 1683;
GALLEX collaboration, *Gallex solar neutrino observations: complete results for Gallex II Phys. Lett.* **B 357** (1995) 237, erratum *ibid.* **B361** (1996) 235.

[6] L. Wolfenstein, *Neutrino oscillations in matter*, *Phys. Rev.* **D 17** (1978) 2369;
S.P. Mikheev and A.Y. Smirnov, *Resonance amplification of oscillations in matter and spectroscopy of solar neutrinos.*, *Sov. J. Nucl. Phys.* **42** (1985) 913; *Resonant amplification of neutrino oscillations in matter and solar neutrino spectroscopy Nuovo Cim.* **9** (1986) 17;

[7] CHOOZ collaboration, *Initial results from the Chooz long baseline reactor neutrino oscillation experiment*, *Phys. Lett.* **B 420** (1998) 397 [hep-ex/9711002].

[8] L. Baudis et al., *Limits on the Majorana neutrino mass in the* $0.1\,eV$ *range*, *Phys. Rev. Lett.* **83** (1999) 41 [hep-ex/9902014].

[9] Particle Data Group, *Review of particle physics*, *Eur. Phys. J.* **C 15** (2000) 1.

[10] Z. Maki, M. Nakagawa and S. Sakata, *Remarks on the unified model of elementary particles Prog. Theor. Phys.* **28** (1962) 870.

[11] M. Fukugita and T. Yanagida, *Unstable neutrinos from the gravitational stellar collapse and Cygnus X-3 Phys. Rev. Lett.* **55** (1985) 2645;
Y. Chikashige, R.N. Mohapatra and R.D. Peccei, *Are there real goldstone bosons associated with broken lepton number?*, *Phys. Lett.* **B 98** (1981) 265;
D. B. Reiss, *Can the family group be a global symmetry?*, *Phys. Lett.* **B 115** (1982) 217;
F. Wilczek, *Axions and family symmetry breaking*, *Phys. Rev. Lett.* **49** (1982) 1549;
G. Gelmini, S. Nussinov and T. Yanagida, *Does nature like nambu-goldstone bosons? Nucl. Phys.* **B 219** (1983) 31;
D.S. Dearborn, D.N. Schramm and G. Steigman, *Astrophysical constraints on the couplings of axions majorons and familons*, *Phys. Rev. Lett.* **56** (1986) 26;
J.L. Feng, T. Moroi, H. Murayama and E. Schnapka, *Third generation familons, b factories and neutrino cosmology*, `hep-ph/9709411`.